# Analysis and control of quality of information (QoI) for wireless sensor networks

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**Abstract:** The Quality of Information (QoI) is an important metric that sensor networks provide to applications. In this paper, a hypothesis testing based framework for sensor networks is considered for analysing the QoI performance under the cases of Single Sensor Single Decision-Making (SSSD), Multi-Sensor Single Decision-Making (MSMD) and Multi-Sensor Multiple Decision-Making (MSMD) detection systems, respectively. The detection probability, false alarm probability and average error probability are treated as the QoI metrics and some explicit solutions are derived, which can capture the impact of various parameters on the QoI metrics. Owing to the dynamic and time-varying characteristics of the target signal of interest, a rate-based QoI control scheme is proposed to effectively control the QoI requirements by adaptively adjusting the data sampling rates of sensors. Numerical results are presented for evaluating the performance of the SSSD, MSSD and MSMD systems and validating the effectiveness of the QoI control scheme.

**Keywords:** quality of information; detection probability; false alarm probability; sampling rate; sensor network.

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# **1** Introduction

Wireless Sensor Networks (WSNs) have recently received significant attention among academia, industry and government agencies due to their great potential in civil and military applications (Tang and Li, 2006). WSNs use spatially distributed sensor nodes to monitor physical or environmental conditions and to cooperatively pass their sensed raw data through the network to a control centre (also called sink node or base station) for decision making. The sink may be able to send commands to sensor nodes for enabling the control of sensor activities. There are various applications in WSNs ranging from healthcare to environment monitoring, from home to industry and from civil to military (Srivastava et al., 2001; Gelenbe and Hey, 2008; Alemdar and Ersoy, 2010; Durisic et al., 2012; Tang, 2013). Different applications drive specific information needs, i.e. the Quality of Information (QoI). There are many definitions of QoI used by researchers. In the work of Sachidananda et al. (2010), the QoI was referred to as the quality experienced/perceived by the user concerning the received information, which (may) fully accomplish the user evolvable requirements while saving valuable resources such as energy and bandwidth. In the work of Charbiwala et al. (2009), the QoI was defined as an application dependent objective function that has a monotonic relationship to the accuracy of the final inference. In the work of Gelenbe and Hey (2008), the QoI was defined as the difference between the data that the output of the WSNs produces concerning some environment that is being monitored and the actual events in that environment which one wishes to observe or track.

Similar to the Quality of Service (QoS) in traditional wireless networks (Lagkas et al., 2010), the QoI is significant in WSNs and is considered as a critical metric that sensor networks provide to applications. The QoI requirement should be ensured regardless of the change of operating conditions of WSNs such as network and environmental conditions, because the sink or decision maker makes decisions based on the confidence it puts on the QoI available to it.

The QoI can be characterised by attributes such as information accurancy, timeliness, reliability, completeness and so on. To determine whether an event of interest has occurred or not, a common way that decision makers use is hypothesis testing. The effectiveness of the test will depend on the information derived through the processing (fusion) of the sensed data and its quality. Thus, for the class of event-detection applications, the two well-known metrics in statistical signal processing (Van Trees, 2001), detection probability and false alarm probability, are suitable to describe the related QoI attributes.

In this paper, we propose a hypothesis testing based framework for WSNs to analyse the QoI metrics (detection and false alarm probabilities and average probability of error) with respect to various parameters such as Signal-to-Noise Ratio (SNR), number of sensors, a priori probability and sensor sampling rates. We analyse our framework under three different scenarios: Single Sensor Single Decision-Making (SSSD) detection system, Multi-Sensor Single Decision-Making (MSSD) system and Multi-Sensor Multiple Decision-Making (MSMD) system. In each case, we derive the explicit solutions of the QoI metrics with respect to related parameters. Based on the analysis of these solutions, we propose a rate-based QoI control scheme to handle the QoI requirements under dynamic and time-varying conditions of the target signal of interest.

The remainder of the paper is organised as follows. Section 2 provides a brief literature survey for different aspects of QoI. Section 3 presents the system description for the detection framework. Section 4 presents the detailed analysis and derives the explicit solutions for the detection system under different scenarios. Section 5 develops a ratebased QoI control scheme. Section 6 presents numerical results and simulations. Finally, the paper is concluded in Section 7.

# 2 Related work

Much research has been done for modelling, analysing and controlling different aspects of QoI on fundamental networking operations such as rate control (Bisdikian, 2007; Fan et al., 2008; He and Zafer, 2008; Charbiwala et al., 2009; Paek and Govindan, 2010), scheduling (Urgaonkar, 2011; Ciftcioglu and Yener, 2011; Ciftcioglu and Yener, 2012; Ciftcioglu et al., 2014), routing (Chu et al., 2002; Liu et al., 2005; Tan et al., 2010) and other techniques and methods (Ronald, 2006; Hunkeler and Scotton, 2008; Tan and Gillies, 2009; Liu et al., 2010). The following gives a brief review.

In the work of Bisdikian (2007), the relationship between the QoI attributes of timeliness and confidence was derived and the dependence of the operational characteristics of sensor systems and the attributes on each other was studied. The analysis in the paper is solid but the shortage is that the considered system comprises only one sensor, which limits its practical applications. In the work of He and Zafer (2008), the problem of interest is the detection of transient signals in Additive White Gaussian Noise (AWGN) in the presence of missing signal observations (samples). A strategy was proposed to adapt the sampling rate in response to missing samples with the goal of achieving accurate and timely decisions with the minimum communication cost measured by sampling rate. In the work of Charbiwala et al. (2009), a centralised rate control mechanism for an event detection scenario was constructed to optimise rate allocation with respect to a QoI metric for transport of sensor measurements in a multi-node multi-hop sensor network. The sensing system considered by Charbiwala et al. (2009) is a centralised multi-sensor system with one fusion centre.

To design a solution for fair and high throughput data extraction from all nodes in the presence of renewable energy sources, Fan et al. (2008) proposed a centralised algorithm and an asynchronous distributed algorithm that can compute the optimal rate assignment for all sensor nodes. In the work of Paek and Govindan (2010), a rate-controlled reliable transport protocol was studied for constrained sensor nodes. To achieve the advantages of efficiency and flexibility, the protocol uses end-to-end explicit loss recovery, but places all the congestion detection and rate adaptation functionality in the sinks.

In the work of Urgaonkar (2011), a QoI aware scheduling was investigated for task processing networks. An optimal scheduling policy was characterised that maximises the average utility delivered by the network. In the work of Ciftcioglu and Yener (2011), QoI aware transmission policies to attain a maximum QoI output utility were considered in the presence of time-varying links in a mobile ad hoc network. In the work of Ciftcioglu and Yener (2012), QoI-based resource allocation in a scenario was considered where multiple reporter nodes send information on an event of interest to a sink node. In the work of Ciftcioglu et al. (2014), a dynamic scheme was formulated for scheduling with the objective of minimising the energy consumption of the network while satisfying constraints on outage probability for QoI.

In the work of Chu et al. (2002), two novel techniques, Information-Driven Sensor Querying (IDSQ) and Constrained Anisotropic Diffusion Routing (CADR), were proposed for energy-efficient data querying and routing in ad hoc sensor networks for a range of collaborative signal processing tasks. In the work of Liu et al. (2005), information-directed routing was formulated with the objective of minimising communication cost while maximising information gain. Tan et al. (2010) studied the problem of finding the least-cost routing tree that satisfies a given QoI constraint.

In the work of Ronald (2006), an analytic framework was presented to study the competition between the negative effect of misassociation and the positive effect of synthesis for demonstrating and analysing their interplay quantitatively. In the work of Hunkeler and Scotton (2008), a framework was presented to process arbitrary sensornetwork data models. Such models can be used to detect anomalies, compress data or combine data from many inexpensive sensors to increase the quality of the measurements. In the work of Tan and Gillies (2009), three potential QoI metrics, i.e. entropy of the co-variance matrix, information gain, residual likelihood, were investigated for estimating the dynamic target tracking performance of systems based on some state estimation algorithms. In the work of Liu et al. (2010), three key design elements were investigated in support of QoI-aware network management of multitasking WSNs: (a) the OoI satisfaction index of a task; (b) the QoI network capacity and (c) an adaptive, negotiation-based admission control mechanism that reconfigures and optimises the usage of network resources in order to optimally accommodate the QoI requirements of all tasks.

#### **3** System description

Consider a sensor network that employs a matched-filter detector for event detection purposes. The detection system consists of two functional subsystems: (a) sensing subsystem comprising U sensor nodes, which samples the environment for an event and sends the samples to the decision maker through white noise; (b) decision making subsystem which processes the corrupted samples and decides whether an event of interest has occurred or not. For a general flat network topology, the decision making subsystem is located in the base station. In some cases, part of the pre-processing function may be implemented in the sensor subsystem. For a hierarchical topology such as a cluster-based sensor detection system, however, the decision making may involve two levels: first, each Cluster Head (CH) makes a decision locally based on the measurements from its own cluster; second, the base station makes the final decision based on the local decisions from all the CHs. We shall study the QoI performance of both the flat network topology and the hierarchical topology in the following analysis.

Assume that the event of interest generates an unknown deterministic signal s(t), as shown in Figure 1. The *j*-th sensor,  $1 \le j \le U$ , samples the signal to get measurements at discrete time instants. Let  $s_{ij}$  be the *i*-th value of s(t)

obtained by the *j*-th sensor at time instant  $t_i$ ,  $1 \le i \le V_j$ ,  $t_i \in [0, T_j]$ , where  $T_j$  is the sensing interval of the *j*-th sensor. The noise n(t) is assumed to be bandlimited Gaussian process of zero mean and Power Spectral Density (PSD)  $\Phi_j(\omega)$  at sensor *j*, which is given by:

$$\Phi_{j}(\omega) = \begin{cases} N_{0j} / 2, & |\omega| < W_{j} \\ 0, & \text{others.} \end{cases}$$





The uncertainty of the original signal s(t) lies in the noise. When an event occurs, the observable outcome of the *i*-th measurement from the *j*-th sensor at time instant  $t_i$ , denoted by  $r_{ij}$ , will comprise both signal and noise components. Thus, during the time interval  $[0, T_j]$ , the number of received samples from the *j*-th sensor is  $V_j$  and the total number of received samples from the *U* sensors is  $\sum_{j=1}^{U} V_j$ .

After the sensing subsystem sends out the collected samples, the decision making subsystem will take an action to decide the occurrence of an event of interest. The accuracy of the action will depend on the QoI that is provided to the decision making subsystem. In a detection system, the two most commonly used performance metrics are the probability of detection (or probability of missed detection) and the probability of false alarm. A missed detection happens when the decision maker incorrectly determines that a target event is absent but the event is actually present. On the contrary, when the decision maker incorrectly determines that a target event is present but the event is actually absent, a false alarm happens. We shall select the detection probability and false alarm probability as the accuracy attribute of QoI for our study.

# 4 QoI analysis

For the purpose of QoI analysis, we start with the simplest scenario, i.e. a SSSD detection system, then we extend it to a MSSD system and finally we present a MSMD system.

# 4.1 Single sensor single decision-making (SSSD)

In order to test the QoI involved hypothesis that a target event occurs (hypothesis H1) or not (hypothesis H0), we assume that during time interval [0, T] there are V (Note in this Section we omit the subscript j for related parameters due to the case of single sensor) independent samples collected by a single sensor and sent to its decision making subsystem under AWGN and consider the following traditional binary hypotheses:

$$H0: r_i = n_i, i = 1, 2, \dots, V$$
(1)

$$H1: r_i = s_i + n_i, \ i = 1, 2, \dots, V$$
(2)

where  $s_i$  represents the value of the signal at the *i*-th sampling instance,  $n_i$  represents the *i*-th sample of AWGN with zero mean and PSD of  $N_0/2$  and  $r_i$  is the observable outcome of the *i*-th measurement at the decision making subsystem. For easy presentation, we re-write the binary hypotheses in vector form:

$$H0: r = n \tag{3}$$

$$H1: r = s + n \tag{4}$$

where vector  $r = [r_1, r_2, ..., r_{V]}^T$ , vector  $s = [s_1, s_2, ..., s_{V]}^T$  and vector  $n = [n_1, n_2, ..., n_{V]}^T$ . Clearly, the vector r is a Gaussian vector with the mean of  $E_0[r] = 0$  under H0 and  $E_1[r] = s$  under H1. The covariance matrix of the AWGN is  $C = E[nn^T]$ . Thus, the following conditional Probability Density Functions (pdf) under H0 and H1 can be, respectively, obtained.

$$f(\mathbf{r}|\mathbf{H}_{0}) = \frac{1}{(2\pi)^{V/2} |\mathbf{C}|^{1/2}} \exp\left[-\frac{\mathbf{r}^{T} \mathbf{C}^{-1} \mathbf{r}}{2}\right]$$
(5)

$$f(\mathbf{r}|\mathbf{H}_{1}) = \frac{1}{(2\pi)^{V/2} |\mathbf{C}|^{1/2}} \exp\left[-\frac{(\mathbf{r}-\mathbf{s})^{T} \mathbf{C}^{-1}(\mathbf{r}-\mathbf{s})}{2}\right]$$
(6)

The likelihood ratio of the conditional probabilities is:

$$\Lambda(\mathbf{r}) = \frac{f(\mathbf{r} \mid \mathbf{H}_{1})}{f(\mathbf{r} \mid \mathbf{H}_{0})} = \exp\left[\mathbf{r}^{T} \mathbf{C}^{-1} \mathbf{s}\right] \exp\left[-\frac{\mathbf{s}^{T} \mathbf{C}^{-1} \mathbf{s}}{2}\right] \sum_{\mathbf{H}_{0}}^{\mathbf{H}_{1}} \lambda_{0} \quad (7)$$

where the term  $\zeta = \mathbf{s}^T \mathbf{C}^{-1} \mathbf{s}$  represents the SNR of the detection system and  $\lambda_0$  is the threshold depending on the selected decision criterion. For example, under a general Bayesian decision criterion  $\lambda_0$  can be derived as (Van Trees, 2001):

$$\lambda_0 = \frac{p(H_0)(C_{10} - C_{00})}{p(H_1)(C_{01} - C_{11})},$$

where p(H0) and p(H1) are the a priori probabilities under hypotheses H0 and H1 and  $C_{00}$ ,  $C_{10}$ ,  $C_{11}$  and  $C_{01}$  represent the cost for the four courses of action, i.e. choose H0 when H0 is true; choose H1 when H0 is true; choose H1 when H1 is true; and choose H0 when H1 is true.

Taking the logarithm on both sides of equation (7) and doing some adjustments, we have:

$$\mathbf{r}^{T}\mathbf{C}^{-1}\mathbf{s}\sum_{H_{0}}^{H_{1}}\ln\lambda_{0} + \frac{\mathbf{s}^{T}\mathbf{C}^{-1}\mathbf{s}}{2}$$
 (8)

In order to find the expression the covariance matrix C, we derive the autocorrelation function R(t) of the bandlimited Gaussian process mentioned in Section 3 as:

$$R(t) = \frac{1}{2\pi} \int_{-W}^{W} \Phi(\omega) \exp(j\omega t) d\omega = N_0 F \operatorname{sinc}(2\pi F t) \qquad (9)$$

where  $F = W/2\pi$  and sinc(x) = sin(x)/x is the cardinal sine function. The first zero crossing of R(t) occurs at time t = 1/(2F). If the sampling rate is selected as 2F, all the noise samples will be uncorrelated, which makes the derivation of an optimal detector very simple. The number of independent observable samples during time interval [0, T] is V = 2FT. The covariance matrix C is a diagonal matrix of order 2FT with the form.

$$\mathbf{C} = N_0 F I \tag{10}$$

where I is the identity matrix of order 2FT, then we have:

$$\mathbf{C}^{-1} = \frac{\mathbf{I}}{N_0 F} \tag{11}$$

Substituting  $C^{-1}$  in equation (8) with equation (11) yields the test statistic as:

$$\gamma \doteq \mathbf{r}^T \mathbf{s} \sum_{\mathbf{H}_0}^{\mathbf{H}_1} N_0 F \ln \lambda_0 + \frac{\mathbf{s}^T \mathbf{s}}{2} \doteq \lambda_0$$
(12)

where  $\gamma_0$  is the decision threshold. Note that the test statistic  $\gamma$  is a Gaussian random variable. Once we obtain its mean and variance, we can determine its pdf.

Under H0, the mean and variance of  $\gamma$  are:

$$E_{0}(\gamma) = E[n^{T}s] = 0,$$
  

$$Var_{0}(\gamma) = E[(n^{T}s)^{T}(n^{T}s)] = N_{0}F(s^{T}s) = N_{0}FE_{s},$$

where  $E_s = \mathbf{s}^T \mathbf{s} = \sum_{i=1}^{2FT} s_i^2$  represents the energy of the signal measured during time interval [0, *T*].

Under H1, the mean and variance of  $\gamma$  are:

$$E_1(\gamma) = E[(s+n)^T s] = s^T s$$
$$Var_1(\gamma) = N_0 F E_s.$$

The pdf of  $\gamma$  under H0 and H1 are, respectively, obtained as:

$$f(\gamma | \mathbf{H}_0) = \frac{1}{\sqrt{2\pi N_0 F E_s}} \exp\left(-\frac{\gamma}{2N_0 F E_s}\right)$$
(13)

$$f(\gamma | \mathbf{H}_{1}) = \frac{1}{\sqrt{2\pi N_{0} F E_{s}}} \exp\left[-\frac{(\gamma - E_{s})^{2}}{2N_{0} F E_{s}}\right]$$
(14)

Thus, the QoI metrics for the SSSD system, i.e. the detection probability  $p_d^{SS}$  and false alarm probability  $p_f^{SS}$  can be derived as:

$$p_d^{SS} = \Pr\left(\gamma \ge \gamma_0 \left| \mathbf{H}_1 \right. \right) = Q\left(\frac{\sqrt{N_0 F} \ln \lambda_0}{\sqrt{E_s}} - \frac{\sqrt{E_s}}{2\sqrt{N_0 F}}\right) \quad (15)$$

Analysis and control of quality of information

$$p_f^{SS} = \Pr\left(\gamma \ge \gamma_0 \left| \mathbf{H}_0 \right. \right) = Q\left(\frac{\sqrt{N_0 F} \ln \lambda_0}{\sqrt{E_s}} + \frac{\sqrt{E_s}}{2\sqrt{N_0 F}}\right) \quad (16)$$

where  $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2}) dt$  is the standard *Q*-function. Note that the SNR is  $\zeta = E_s / (N_0 F)$ , we can re-write  $p_d^{SS}$  and  $p_f^{SS}$  as follows:

$$p_d^{SS} = Q\left(\frac{\ln\lambda_0}{\sqrt{\zeta}} - \frac{\sqrt{\zeta}}{2}\right)$$
(17)

$$p_f^{SS} = Q\left(\frac{\ln\lambda_0}{\sqrt{\zeta}} + \frac{\sqrt{\zeta}}{2}\right) \tag{18}$$

Since Q(x) is monotonically decreasing of x, it has a welldefined inverse  $Q^{-1}(\cdot)$ . This property allows the Q-function related metrics to be widely used in system analysis and design. In the applications of signal detection, it is usually focused on analysing the performance of one metric given the constraint of the other. For example, according to Neyman-Pearson decision criterion, given an upper bound value of  $p_f^{SS}$ , i.e.  $p_f^{SS} \le p_{f_0}^{SS}$ , we can find the maximum value of  $p_d^{SS}$ :

$$p_{d(\max)}^{SS} = Q\left(Q^{-1}\left(p_{f_0}^{SS}\right) - \sqrt{\zeta}\right)$$
(19)

# 4.2 Multi-sensor single decision-making (MSSD)

In practical sensing detection systems, a large portion of applications involve multiple sensors. Now we extend our QoI analysis of SSSD system to an MSSD system, which can be applied to the flat sensor network topology. We assume that there are a total of  $\sum_{j=1}^{U} V_j$  independent samples collected by *U* sensors and sent to a single decision making subsystem, where  $V_j$  is the number of samples collected by sensor *j* during time interval  $[0, T_j], V_j = 2F_jT_j, F_j = W_j/2\pi, 1 \le j \le U$ . Correspondingly, we use  $s_{ij}$  to denote the *i*-th value of s(t) obtained by the *j*-th sensor at time instant  $t_i, 1 \le i \le V_j$ . Assume that the noise is independent across samples and sensors. Then, the covariance matrix **C** is a diagonal matrix of order  $\sum_{j=1}^{U} V_j$  with the form

$$\mathbf{C} = diag(N_{01}F_{1}\mathbf{I}_{V_{1}}, N_{02}F_{2}\mathbf{I}_{V_{2}}, \cdots, N_{0U}F_{U}\mathbf{I}_{V_{U}}).$$

Then, following the same procedure of sub-section 4.1, we derive the QoI metrics of the MSSD system,  $p_d^{MS}$  and  $p_f^{MS}$ , as follows:

$$p_d^{MS} = Q\left(\frac{\ln\lambda_0}{\sqrt{\zeta_{MS}}} - \frac{\sqrt{\zeta_{MS}}}{2}\right)$$
(20)

$$p_f^{MS} = Q\left(\frac{\ln\lambda_0}{\sqrt{\zeta_{MS}}} + \frac{\sqrt{\zeta_{MS}}}{2}\right)$$
(21)

where the SNR  $\zeta_{MS}$  has been changed to

$$\zeta_{MS} = \sum_{j=1}^{U} \frac{1}{N_{0j} F_j} \sum_{i=1}^{V_j} (s_{ij})^2$$
(22)

#### 4.3 Multi-sensor multiple decision-making (MSMD)

Alternative to the MSSD system, there are some applications using multiple decision makers. Such MSMD systems are usually applied to a hierarchical sensor network topology such as a cluster-based sensor network. For a basic twolevel hierarchical topology, the first level of decision maker (e.g., a cluster head) makes a decision locally based on the measurements from its own cluster; then, the second level of decision maker (e.g., base station) makes the final decision based on all the local decisions from the first level.

Consider the following cluster-based WSN scenario where there are a total of U sensors located in M clusters,  $M \le U$ ; each cluster k is operated by a cluster head  $(CH_k)$ that manages  $U_k$  sensors of its cluster (including itself), i.e.  $U = \sum_{k=1}^{M} U_k$ . Without loss of generality, assume that each CH does the sensing task besides the decision making task. The M CHs do the first level decisions and send their results to the base station, which makes the final decision.

Let  $p_f^{CH_k}$  and  $p_d^{CH_k}$  be the false alarm probability and detection probability at cluster *k*, respectively. Clearly, the QoI metrics at each cluster are the same as that of the MSSD system. Thus, we can directly obtain them as follows:

$$p_{d}^{CH_{k}} = Q\left(\frac{\ln\lambda_{0}}{\sqrt{\zeta_{CH_{k}}}} - \frac{\sqrt{\zeta_{CH_{k}}}}{2}\right)$$
(23)

$$p_f^{CH_k} = Q\left(\frac{\ln\lambda_0}{\sqrt{\zeta_{CH_k}}} + \frac{\sqrt{\zeta_{CH_k}}}{2}\right)$$
(24)

where the SNR  $\zeta_{CH_k}$  is obtained as:

$$\zeta_{CH_k} = \sum_{j=1}^{U_k} \frac{1}{N_{0jk} F_{jk}} \sum_{i=1}^{V_{jk}} (s_{ijk})^2$$
(25)

and  $V_{jk}$  is the number of samples collected by the *j*-th sensor at cluster *k* during time interval  $[0, T_{jk}]$ ,  $V_{jk} = 2F_{jk}T_{jk}$ ,  $1 \le j \le U_k$ ;  $S_{ijk}$  is the *i*-th value of s(t) obtained by the *j*-th sensor of cluster *k* at time instant  $t_i$ ,  $1 \le i \le V_{jk}$ ;  $N_{0jk}$  is the single sided PSD at sensor *j* of cluster *k*.

The base station fuses the local decisions of all M clusters and makes the final decision according to a certain fusion rules such as OR-rule and AND-rule (Varshney, 1997). In the OR-rule, the base station receives the decisions from the M CHs and decides H1 if any of the M individual decisions is H1. In the AND-rule, the base station decides H1 only if all M individual decisions are H1. This former fusion rule can lead to a high detection probability, but also a high false alarm probability. The latter rule goes to the other extreme edge (i.e. low detection and low false alarm probabilities).

#### 108 S. Tang and B. Davis

Here we choose a trade-off fusion scheme for analysis, called *K*-out-of-*M* fusion rule, which was originally widely used in Reliability engineering (Birnbaum et al., 1961). In the *K*-out-of-*M* rule, the base station decides H1 if and only if at least *K* of the *M* individual decisions are H1.

Let  $p_f^{MM}(K,M)$  and  $p_d^{MM}(K,M)$  be, respectively, the false alarm probability and detection probability of the MSMD system according to the *K*-out-of-*M* rule. Then,  $p_d^{MM}(K,M)$  and  $p_f^{MM}(K,M)$  can be calculated by:

$$p_{d}^{MM}(K,M) = \sum_{k=K}^{M} \binom{M}{k} \prod_{i=1}^{k} p_{d}^{CH_{i}} \prod_{j=k+1}^{M} \left(1 - p_{d}^{CH_{j}}\right)$$
(26)

$$p_{f}^{MM}(K,M) = \sum_{k=K}^{M} \binom{M}{k} \prod_{i=1}^{k} p_{f}^{CH_{i}} \prod_{j=k+1}^{M} \left(1 - p_{f}^{CH_{j}}\right)$$
(27)

where  $\binom{M}{k} \stackrel{\circ}{=} \frac{M!}{k!(M-k)!}, \prod_{j=M+1}^{M} (\cdot) \stackrel{\circ}{=} 1$ . To reduce the

computational complexity of the two probabilities  $p_f^{MM}(K,M)$  and  $p_d^{MM}(K,M)$ , the following recursive formula is often used.

$$p_{x}^{MM}(K,M) = \left(1 - p_{x}^{CH_{M}}\right) p_{x}^{MM}(K,M-1) + p_{x}^{CH_{M}} p_{x}^{MM}(K-1,M-1), \quad x \in \{f,d\}$$
(28)

In a special case of equations (26) and (27) where the *M* CHs are assumed to use the same decision rule (i.e. the same threshold) and to experience independent and identically distributed noise with the same SNR (Ghasemi and Sousa, 2005), we shall have the same probability for all CHs, i.e.  $p_d^{CH_k} = p_d$ ,  $p_f^{CH_k} = p_f$ ,  $1 \le k \le M$  and further obtain:

$$p_{d}^{MM}(K,M) = \sum_{k=K}^{M} {\binom{M}{k}} (p_{d})^{k} (1-p_{d})^{M-k}$$
(29)

$$p_{f}^{MM}(K,M) = \sum_{k=K}^{M} {\binom{M}{k}} (p_{f})^{k} (1-p_{f})^{M-k}$$
(30)

The value of *K* can be flexibly adjusted for different system requirements. The above formulas include the following special cases: (a) when K = 1, the *K*-out-of-*M* rule becomes the OR-rule; (b) when K = M, the *K*-out-of-*M* rule becomes the AND-rule; (c) when K = [M/2], where [x] denotes the smallest integer not less than x, equations (29) and (30) are the majority rule, i.e. a candidate wins with a majority (more than 50%) of voters.

# 5 A rate-based QoI control scheme

In reality, the observations of the target signal collected often shows dynamic and time-varying characteristics and the noise environment is also changeable to sensors due to wind, rain, flying birds, engine-related interference and other man-made factors. The configuration of QoI metrics initially designed for a sensing detection system may not always be satisfied. Thus, an appropriate QoI control scheme should be employed for sensor detection systems to maintain the QoI requirements under dynamic conditions.

Consider a MSSD system as an example for developing the QoI control scheme. We describe the QoI through a single metric involving the two probabilities, the average probability of error  $P_e$ . Our objective is to propose a QoI control scheme for building the relationship between the QoI metric  $P_e$  and the sampling rates of individual sensors, i.e. a rate-based QoI control scheme.

We assume that there is no cost for a correct decision and that the cost for a wrong decision is equal to 1, i.e.  $C_{00} = C_{11} = 0$  and  $C_{01} = C_{10} = 1$ , then the average error probability for the MSSD system is obtained as:

$$P_{e} = p(\mathbf{H}_{0})p_{f}^{MS} + p(\mathbf{H}_{1})(1 - p_{d}^{MS})$$
$$= p(\mathbf{H}_{0})Q\left(\frac{\ln\lambda_{0}}{\sqrt{\zeta_{MS}}} + \frac{\sqrt{\zeta_{MS}}}{2}\right) + p(\mathbf{H}_{1})Q\left(\frac{\sqrt{\zeta_{MS}}}{2} - \frac{\ln\lambda_{0}}{\sqrt{\zeta_{MS}}}\right)$$
(31)

where  $\lambda_0 = p(H0)/p(H1)$ , the a priori probabilities p(H0) and p(H1) can usually be estimated through the collected plenty of empirical data from field experiments. Notice that the average error probability  $P_e$  is a function of the SNR,  $\zeta_{MS}$ , i.e.  $P_e$  is monotonically decreasing for all values of  $\zeta_{MS}$  (which can be verified in Figures 3 and 4 in the next section). From equation (22), we know that  $\zeta_{MS}$  is a function of the number of samples collected by all the sensors.

Let  $\beta_j$  be the sampling rate that sensor *j* employs during  $[0, T_j]$  and *B* be the sampling rate vector of the whole MSSD system, then *B* can be written as  $B = [\beta_1, \beta_2, ..., \beta_U]^T$ . For simplicity, we further assume that all the *U* sensors use the same sampling time interval, i.e.  $T_1 = T_2 = ... = T_U = \tau$ . Then, we have  $V_j = \beta_j \cdot \tau$ ,  $1 \le j \le U$  and equation (22) can be written as:

$$\zeta_{MS}(\mathbf{B}) = \sum_{j=1}^{U} \frac{1}{N_{0j} F_j} \sum_{i=1}^{\beta_j \tau} (s_{ij})^2 = \sum_{j=1}^{U} \zeta_j(\beta_j)$$
(32)

where  $\zeta_j(\beta_j)$  represents the individual sensor level SNR that contributes to the system level SNR. The larger the sampling rate  $\beta_j$ ,  $1 \le j \le U$ , the higher the SNR  $\zeta_j(\beta_j)$  and thus  $\zeta_{MS}(\mathbf{B})$ . Thus, our problem of QoI control can be formulated as:

$$\arg\min_{\mathbf{p}} P_e\left(\zeta_{MS}(\mathbf{B})\right) \tag{33}$$

Since the probability  $P_e$  will decrease when B is increased, an appropriate rate-based QoI control scheme can be designed to decrease  $P_e$  through increasing the sampling rate vector B of the MSSD system. However, the increase of sampling rates means more samples to be sent to the base station and thus more energy consumed, though the energy issue is not yet considered in our framework. Thus, in the design of the QoI control scheme, the mechanism of decreasing the sampling rates at the condition of very low  $P_e$  should also be considered.

The algorithm of the rate-based QoI control scheme is shown in Figure 2, where  $B_0 = [\beta_{10},\beta_{20}, \ldots,\beta_{U0}]^T$  denotes the initial sampling rate vector assigned,  $\Delta = [\delta_1, \delta_2, \ldots, \delta_U]^T$  denotes rate adjustment vector which depends on specific applications and can be obtained through empirical measurements and  $P_{e(max)}$  and  $P_{e(min)}$  are the two predefined thresholds of the average error probability for triggering the increase and decrease of the rate vector *B*. Clearly, the same approach can be applied to the MSMD system.

Figure 2	The algorithm	of the rate-based	OoI control scheme

QoI Rate Control: Initialization: U, F,  $\tau$ ,  $B_{\theta}$ ,  $\Delta$ ,  $p(H_0)$ ,  $P_{e(max)}$ ,  $P_{e(min)}$ . **1**: Collect samples from each sensor j, 1 = j = U, during a sampling interval  $\tau$  and calculate  $P_e$ according to equations (31) and (32) at the sink. **2**: Sink compares  $P_e$  with a predefined threshold  $P_{e(th)}$ . If  $P_e > P_{e(max)}$ , { Sink broadcasts rate update (increase) to all the sensors " $\beta_i$ ? min{ $\beta_{j(max)}$ ,  $\beta_j + \delta_j$ }, 1 = j = U"; (Note: Initially  $\mathbf{B}$ ?  $\mathbf{B}_0$ ) go to  $\mathbf{3}$ elseif  $P_e < P_{e(min)}$ , { Sink broadcasts rate update (decrease) to all the sensors " $\beta_j$ ? max {  $\beta_{j(min)}$ ,  $\beta_j$  -  $\delta_j$  }, 1 = j = U"; go to  $\mathbf{3}$ else {No broadcasting; go to 1 } **3**: Each sensor updates its rate  $\beta_i$  for signal sensing; go to 1.

# 6 Numerical results

In this section, we first present numerical results for the performance of SSSD, MSSD and MSMD systems by studying the QoI metrics with respect to various parameters and then provide simulations for the rate-based QoI control scheme. The parameter configuration is as follows. The a priori probability p(H0) is set as 1/2 otherwise specified. The threshold is  $\lambda_0 = p(H0)/p(H1)$ . The number of sensors U is set as 1, 5 and 10, respectively, for the SSSD and MSSD systems. For the MSMD system, a total of U = 20 sensors are evenly distributed in M = 5 clusters. The change of SNR is displayed on separate figures.

Figure 3 shows how the QoI metric of the SSSD system  $p_d^{SS}$  changes with respect to the change of SNR  $\zeta$  and the probability p(H0). As expected, when  $\zeta$  is increased,  $p_d^{SS}$  will increase as the signal of higher SNR is easier to be detected. We observe that the increase of p(H0) will lead to the decrease of  $p_d^{SS}$ , since the increase of p(H0) is equivalent to the decrease of p(H1), which probably reduces the opportunity of detection (decision of H1).





Figure 4 shows how the QoI metric  $p_f^{SS}$  changes with respect to the change of  $\zeta$  and p(H0). For the same reason,  $p_f^{SS}$  decreases when  $\zeta$  is increased. We also observe when the p(H0) is increased,  $p_f^{SS}$  will decrease, since at this case the decision in favour of H1 becomes more difficult.



Figure 5 shows the ROC curves at different SNR conditions. As expected, the higher the value of  $\zeta$ , the higher the detection probability  $p_d^{SS}$  for a given false alarm probability  $p_f^{SS}$ ; alternatively, the lower the  $p_f^{SS}$  for a given  $p_d^{SS}$ .

Figures 6 and 7 shows the change of QoI metrics of the MSSD system  $p_d^{MS}$  and  $p_f^{MS}$  with respect to the change of  $\zeta$  and the number of sensors *U*. A curve for the SSSD system (U = 1) is provided for comparison. We observe that the performance of QoI significantly improves in the multiple-sensor condition, i.e. detection probability  $p_d^{MS}$  becomes higher and false alarm probability  $p_f^{MS}$  becomes lower.

**Figure 5** QoI performance: ROC curves ( $p_d^{SS}$  vs.  $p_f^{SS}$ ) under various SNR values (see online version for colours)



**Figure 6** QoI performance:  $p_d^{MS}$  and  $p_d^{SS}$  vs. various parameters (p(H0) = 0.5) (see online version for colours)



Figure 7 QoI performance:  $p_f^{MS}$  and  $p_f^{SS}$  vs. various parameters (p(H0) = 0.5) (see online version for colours)



Figures 8 and 9 shows the change of QoI metrics of the MSMD system  $p_d^{MM}$  and  $p_f^{MM}$  with respect to the change of various parameters. In the calculation, we consider that the multi-sensor system consists of M = 5 clusters, each have four sensors. We compare the performance of QoI metrics  $p_d^{MM}$  and  $p_f^{MM}$  are compared under OR-rule (K = 1) and-rule (K = 5) and the majority rule (K = 3), respectively. As a comparison, we also present the metrics of the MSSD system in both figures. It can be observed that the OR-rule is in favour of high detection probability  $p_d^{MM}$ , but it simultaneously brings a significantly high false alarm probability  $p_f^{MM}$ . The AND-rule produces a very low  $p_d^{MM}$ , although it has a very low  $p_d^{MM}$ .

**Figure 8** QoI performance:  $p_d^{MM}(K,M)$  and  $p_d^{MS}$  vs. various parameters (p(H0) = 0.5) (see online version for colours)



**Figure 9** QoI performance:  $p_f^{MM}(K,M)$  and  $p_f^{MS}$  vs. various parameters (p(H0) = 0.5) (see online version for colours)



On the contrary, the *K*-out-of-*M* rule can achieve a good balance for the  $p_d^{MM}$  and  $p_f^{MM}$ . For different requirements of  $p_d^{MM}$  and  $p_f^{MM}$ , we can select a proper *K* for the *K*-out-of-*M* scheme. The performance of the *K*-out-of-*M* scheme can approach that of the MSSD system, as seen in Figures 8 and 9. The MSSD system, as the former utilises all the available information to the highest degree. This can also be verified by the average error probability  $P_e$  in Figure 10, where the MSSD system achieves the smallest  $P_e$ . The MSMD system with the majority rule can approach that minimum, while the MSMD system with the OR-rule or AND-rule has much larger  $P_e$ .

Figure 10 QoI performance:  $P_e$  of the MSMD and MSSD systems vs. various parameters (p(H0) = 0.5) (see online version for colours)



The above figures presented the performance of QoI metrics for the SSSD, MSSD and MSMD systems. Considering the variability of the SNR of the target signal, next we perform simulations by MATLAB to verify the feasibility of the proposed rate-based QoI control scheme. The simulation scenario is built by considering a MSSD system with a total of U = 20 sensors under AWGN environment and a repeatedly occurred target signal s(t) with negative exponential distribution (exponentially decaying), given by:

$$s(t) = a \cdot e^{-bt}, \quad a > 0, b > 0.$$

The related parameter configuration is setup as follows:  $N_0 = 5$ , F = 1,  $\tau = 5$ , a = 3, b = 0.3, p(H0) = 0.5. All the sensors are identical with the same initial sampling rate  $\beta$ and the same adjusted quantity  $\delta$ . The predefined threshold of error probabilities are  $P_{e(max)} = 10\%$ ,  $P_{e(min)} = 2\%$ . We simulate the target signal occurs repeatedly with a period of 10 time units to verify the effectiveness of the rate-based QoI control scheme.

The detailed dynamic behaviours of the simulation executions are shown in Figures 11 and 12, where the error bars show 95% confidence intervals obtained by running 1,000 simulation trials for each point. Figure 11 shows the simulation of the change of  $P_e$  in the MSSD system without applying the QoI control scheme. The  $P_e$  becomes gradually

increasing in each occurrence period owing to the target signal decay. The line at the  $P_e$  of 10% indicates the predefined threshold of  $P_{e(max)}$  (upper bound). It can be seen that the error probability  $P_e$  increases and crossovers the upper bound in every occurrence period.

Figure 11 Simulation of the QoI metric  $P_e$  under varying noise without rate control (see online version for colours)



Figure 12 Simulation of the QoI metric  $P_e$  under varying noise using rate control (see online version for colours)



Figure 12 shows the simulation of the change of  $P_e$  by applying the QoI control scheme under the same condition of Figure 11. In the first period of signal occurrence, the QoI control does not take effect probably because the simulation needs some time to be stable. We observe that when  $P_e$  gradually increases and exceeds the predefined threshold  $P_{e(max)}$ , the QoI control scheme is triggered and appropriate operations are performed in accordance with Figure 2 and then  $P_e$  becomes decreasing until below the threshold  $P_{e(max)}$  due to the increase of sampling rates. The process repeats in the subsequent other occurrence periods, where in the initial simulation time,  $P_e$  becomes very low due to the usage of both updated (high) sampling rates and relatively large values of observations. Then the QoI control scheme is again triggered and  $P_e$  increases until it is above

the threshold  $P_{e(min)}$  (The line at the  $P_e$  of 2% is not shown in Figure 12). This experiment demonstrates that the rate-based QoI control scheme is feasible.

# 7 Conclusions

We proposed a hypothesis testing based framework for WSNs to analyse the QoI metrics (detection and false alarm probabilities and average probability of error) under three different scenarios: SSSD detection system, MSSD system and MSMD system. We derived the explicit solutions of the QoI metrics with respect to various parameters such as SNR, number of sensors, a priori probability and sensor sampling rate. Based on the analysis of these solutions, we proposed a rate-based QoI control scheme to adaptively adjust the QoI requirements under dynamic and time-varying conditions of the target signal of interest. The proposed framework and analysis method can be applied to different types of sensor detection systems.

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