Alchian & Allen Theorem

An Application of the First Law of Demand to Product Quality.

\[
\frac{(P_h + c)}{(P_l + c)}
\]

Proved in general from:

\[ P_{full} = P_{market} + P_{transport} \Rightarrow \text{Full Price} = \$ \text{Cost (i.e. Nominal)} + \text{Time Cost.} \]

Two grades of good:
H-high – Price is \( P_h \)
L-low – Price is \( P_l \)

Driven by idiosyncratic variances:
\[ \frac{P_h}{P_l} > 1 \] (high quality product is more valuable)

\[
\frac{(P_h + \text{Fixed Cost})}{(P_l + \text{Fixed Cost})} < \frac{P_h}{P_l}
\]

\[
\frac{(P_h + F)}{(P_l + F)} \text{ after tax} < \frac{P_h}{P_l} \text{ pre tax}
\]

To use the Alchian/Allen Theorem the fixed cost must be equal and added to both prices.

Modified Consumer Theory
\[ U = U(X_i) \] is utility representing anyone of us with any good \( i \).

Now we can rewrite this utility with demanded good \( i \) in terms of service demanded and time

\[ U = U(Z_i, t), \text{ where } Z_i \text{ is services provided by } X_i. \text{ Multiple way to obtain } (Z_i) \]

Make – Buy decision (allocate household production is our objective)

Assume fixed proportions in production (constant ratios of \( Z_i, T_i \))
\[ Z_i = a_i X_i \]

\( a_i \) is a technical coefficient. (how much \( Z_i \) you get from it), which may vary across people. Some people with a lot of human capital have large \( a_i \) and idiots small \( a_i \).

\( X_i \) is something good used to make \( Z_i \).

\[ Z_i = b_i * T_i \] (given amount of time in given amount of output)

For multiple \( X \) we have \( Z_i = \sum_{i=1}^{M} a_i X_i \)

\( b_i \) vary across individuals (one person can take a little time and get a lot of output) – it is idiosyncratic personal product coefficient

\[ \sum_{i=1}^{M} P_i X_i + \sum T_i w = T_w w + g \]

\( \sum_{i=1}^{M} P_i X_i \) is total expenditure on goods with \( P_i \) being price and \( X_i \) being input.

\( \sum T_i w \) is value of the time spent on processing \( X_i \)

\( T_w w + g \) is money income, where \( T \) is time spent working and \( w \) is a paycheck and \( g \) is external sources (unearned income)

Confusion is an adrenaline for Economics Professor.

\[ \alpha_i = \frac{1}{a_i} \] lead to the full price \( \prod_i = \sum_{i=1}^{N} \alpha_i p_i z_i + \sum_{i=1}^{N} \beta_i w z_i \)

\[ \prod_i = Z_i (\sum_{i=1}^{N} \alpha_i p_i + \sum_{i=1}^{N} \beta_i w) \] where \( \sum_{i=1}^{N} \alpha_i p_i \) is money price plus \( \sum_{i=1}^{N} \beta_i w \) time cost

\( Z_i = b_i + T_i \) for example 3 (units)=3*1 or 4(units)=4*1, this person with more skills is better off) If wage, \( w \) is high then the value of time is high.